Connecting students to mathematics as a modern human endeavour

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Introduction and Context

A common perspective among students from traditional Western secondary school education systems is that mathematics is a static tool for modelling and problem solving (Petocz et.al, 2007). Nowhere in the VCE Study Design (VCAA, 2022) is there mention of mathematics as being a modern human activity, complete with cultural norms, that can be undertaken purely for its own sake. A survey of learning outcomes in Level 1 mathematics subjects in science degrees at a sample of Australian Universities yields no examples of subjects for which these ideas appear as a learning outcome. This is despite the fact the Australian Council of Deans of Science lists "explaining the ways in which science is a social endeavour that influences and is influenced by society" as a threshold learning outcome for science graduates in Australia (ACDS, 2023). This limited student perspective is potentially further entrenched by the usual nature of mathematics assessment. Space is rarely given in assessment to activities that engage students on issues related to the culture of mathematics. Within this educational paradigm it would be no surprise for students to complete their studies in a science degree without appreciating that modern mathematics is an active human endeavour, complete with cultural norms.

Intervention

During the COVID-19 pandemic, many approaches were proposed to modify assessment in recognition of the extraordinary circumstances limiting opportunities for traditional assessment. One such proposal was given by Professor Francis Su (2020). His work inspired our modification of the format of the traditional problem set to give a consistent space across a semester for students to engage in reflective thinking about the nature of mathematics. In practice we group these opportunities as a "Part A", and retain the traditional problem set approach as "Part B". The figure below shows how this approach fits into traditional assessment design in mathematics.

Within the approach, we choose reflective activities to connect with the cohort in the subject. The two examples shown below outline the breadth of applicability for this approach for different cohorts. The example on the left was given to a Level 2 subject in Real Analysis. The example on the right was given to students enrolled in a University mathematics bridging program for Indigenous students.

Part A Prompts

PART A
In the place where you accessed this assignment there is a file named *Why Isn't 1 Prime?*. Read the article and write a short response (<250 words) that considers the following questions:
- Was there anything about this reading that you found particularly surprising or interesting?
- Do you think most non-math people would be surprised to find out 1 is not prime?
- Do you think it is a problem that mathematicians and non-mathematicians might use the word prime to mean slightly different things?

A prompt for students learning about proof (left)

PART A
Below is a link to a video recording of a public lecture on Mathematics and Culture by Professor Chris Matthews, the founding Chair of the Aboriginal and Torres Strait Islander Mathematics Alliance (ATSIMA).
You can watch as much of this recording as you would like, but in particular please watch Prof Matthews' explanation of the 'Goompi model'.
Reflecting on Prof Matthews' ideas, write a short response (<250 words) that addresses one or more of the following:
- What are your thoughts about the Goompi model, and the idea that mathematics is a creative process, containing cultural bias just like art or music?
- Has your prior experience of learning maths embraced creativity? Or has it been mainly in the 'cloud' (i.e. abstract) part of the model? How have you found it?
- Can you think of any ways in which maths could be taught differently in order to better connect with your cultural identity? Or simply to better embrace cultural diversity overall?

A prompt for students taking their first steps in tertiary mathematics (right)

Assessment Redesign

Reflective work integrates naturally with the standard design of the problem set in mathematics subjects

Part A in Action

The Goompi model made me think a little different about math. I've always seen it as something fixed and universal, but Matthew's ideas about math, like art or music comes from culture and creativity. This makes sense when looking at traditional methods I have been exposed to. Where math is used for navigation, astronomy and land management in ways that feel connected to real life.

In my own experience, math has mostly been about formulas and abstract ideas. The "cloud". This often felt disconnected from real life, and I didn't always see how it applied to my life. Creativity isn't really encouraged...

Thinking about math as something shaped by culture makes it feel more meaningful...and easier to connect with while also showing math isn't just about numbers. Its about how people understand and interact with the world.

A proof is an argument used to convince an audience of the truth of a particular statement. In Mochizuki's case, he sought to convince other number theorists (and mathematicians in general) of the *abc* conjecture. It seems he has failed to do so, and therefore has failed to prove anything. Despite this, there is a set of 'proofs', which while may fail to convince others of a statement's truth, contain no logical flaws, and must be true. Mochizuki's proof may well fall into this category. While it may contain no concrete flaws, it fails to be of any use to mathematicians if there is misunderstanding and skepticism surrounding it.

The value in proving statements with no obvious non-mathematical use should not be diminished. The concept of complex numbers, first proposed to be solutions to certain types of previously unsolvable polynomial equations, turned out to have uses in uncountable numbers of fields. Much of linear algebra was developed independent of any perceived use. While practical uses for theorems in number theory may seem less obvious, the joy of mathematics shouldn't be limited to what is useful. Hell, could you imagine what great things humanity could do if we applied the mind of Terence Tao to curing cancer, or solving the issue of climate change. But mathematics (and its great minds) seemingly do not care what uses (or lack there of) for their work exist.

I was surprised to learn that 1 is not a prime. Previously, when posed with the question I have immediately or shortly afterwards disregarded the question as I didn't understand why it is important to make such a distinction. Additionally, on occasion I have thought about it and leant towards the 1 is prime side because I didn't make the distinction between being divisible by 1 and itself. For this reason, I think that non-maths people would be surprised to learn that 1 is not a prime.

I am unsurprised to discover that this question has been discussed over centuries. I already knew that 1 was not considered a number for centuries, but it was pleasant to gain some insight into the progression from not being considered a number to being considered a number. Finally, I have always wondered about primes in sets other than N, so I have valued exposure to this discussion of primes, irreducibility, and the definition of a unit of measure greatly.

I think this video has definitely made me rethink my attitude towards making mistakes. I have often seen mistakes as something to avoid – a sign of failure. Boalers explanation that mistakes spark brain growth and deepen learning. It reframed that Mistakes are not setbacks but opportunities.

At the beginning of the unit, my view on mathematics was narrow. I saw it as numbers, equations, solutions, and problem-solving, but throughout the unit, I have gained a greater appreciation for mathematics and its broader role in describing the physical world. I now recognise it as a language that reflects deeper truths about reality and expresses underlying constructs of nature.

Mathematical and scientific knowledge system allows humans to measure and predict aspects of fire behaviour (shape and spread). This assists the planning when a said fire would come, like evacuations and ensuring the right resources. In contrast to Indigenous knowledge which is much more holistic and built over thousands of years, through observation, practice and oral communication.

...I think Indigenous knowledge should be leading the framework for how, when and why fire is used, whilst Westernised science and maths have the tools to extend and build on Indigenous practices. There is a strong scope for mutual leaning via collaboration that's built on respect between Indigenous and Westernised knowledge. As Westernised knowledge provides mathematical modelling tools, Indigenous knowledge guides fire use.

Previously, in Assignment Number 4, we were asked to identify maths in nature. After reading the article "Is Mathematics Man-Made or Intrinsic to Nature?" this raises the question: while many things appear to have mathematical associations, is it possible that these serve a deeper purpose greater than mathematical design?

[The] underlying principles and patterns found in nature predate human cognition, so they are not solely a man-made construct. Democritus, an ancient Greek philosopher, proposed the earliest concept of the atom using the analogy of dust motes in a sunbeam. He observed dust particles moving around in the air when illuminated by a sunbeam and proposed the idea of atoms in constant motion. This idea was based on observation and philosophical reasoning using thought experiments relating to phenomena like the way objects move, as opposed to mathematical calculations.

This suggests that nature follows rules, and as humans, we have developed a language we use to help us understand and make sense of the logic that already exists in nature. While I acknowledge that mathematics is both man-made and innate to nature, there is an important distinction between the two. This leads me to conclude that mathematics didn't develop 'out of nowhere,' but was discovered in nature, and we have developed a language to describe what already exists.

Student responses to Part A prompts

Reflections

To date, this approach has been taken in seven undergraduate mathematics subjects, reaching more than 2000 students. This activity is often mentioned by students during subject surveys as an interesting and thought-provoking part of the subject. It is clear from our experiences that students find value in undertaking this work and quickly come to view this work as a usual part of their formative assessment in the subject.

Students consistently undertake this work authentically with the same (or even greater) care given to the traditional assessment tasks, regularly display nuanced insight and make connections between the reflective work and their studies in mathematics.

For students in minority or underrepresented groups, such as Indigenous students, tasks can be chosen to help address the cultural disconnect often felt around mathematics.

Tasks highlighting the human and cultural aspects of mathematics have helped alleviate 'maths anxiety' in many students, breaking down perceived barriers to success.

Importantly, since reflective tasks seek personal insights and points of view, they show students that their individual creativity and ideas are valued by subject staff. At the same time staff get to know their students better.

Future Work

The work here represents the early stages of a research study to assess the effectiveness of the approach as a mechanism to affect students' perceptions of mathematics as a human endeavor and whether these perceptions impact student success rates and orientation toward future studies in mathematics. Data collection for this research is currently underway.

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