MAST10005 Assignment 3
Semester 2 2025

Due Date: Wednesday September 10

Assignments are due in Gradescope at 10:00pm on the due date listed above. Late assignments are not accepted. Please see the instructions on the page where you accessed this file for details on submitting your assignment in Gradescope.

Students are encouraged to work together on understanding the problems and their solutions. However, submitted solutions must be prepared individually, in your own words, and without the aid of others or materials prepared by others. For example, if you are asked to give an example as part of your solution, your example should be meaningfully different from anyone you have worked with on understanding the problems.

Each question has an annotation, which tells you how you may use generative AI tools (e.g., ChatGPT) as part of your work. The legend can be found in the place you downloaded this file.

If you do not completely understand your submitted solutions, it is possible you are committing academic misconduct.

Please do not upload this assignment paper or any of its questions/solutions to any online *"help"* service (e.g., CourseHero, Chegg, etc...). By doing so, you are actively ruining the learning experience for future students.

(GI:2) PART A

Read the article at: https://www.quantamagazine.org/the-simple-math-behind-the-mighty-roots-of-unity-20210923/

This article discusses solutions to particular complex polynomial equations; namely equations of the form $x^n - 1 = 0$. Solutions to this equation are nth roots of 1 and are called *nth roots of unity*.

Based on your reading of this article, write a typed response (≈ 150 words) that considers the following questions:

- Mathematics has a reputation of being purely logical and objective. Does it surprise you to know that mathematicians sometimes describe mathematical ideas and arguments using words like *elegant* and *beautiful*? What do you think it means for a mathematical idea or argument to be *elegant* or *beautiful*?
- The article describes an *"elegant algebraic argument"* that proves that the sum of the nth roots of unity equals 0. Do you agree the argument is *elegant*?
- Are there any arguments or ideas you've seen in MAST10005 this semester that you would describe as *elegant*? *beautiful*?

(There are lots of great Quanta Magazine articles about complex numbers. If you are interested in physics consider reading the article named *Imaginary Numbers May Be Essential for Describing Reality.*)

PART B

(1) (GI:0) Question (1) on this assignment is a Webwork question. Complete this question by logging on to WebWork on Canvas via the page on which you found this file.

(2) (GI:1)

(a) Use your solution from Problem 2 in WebWork to find all complex solutions of $(z-2)^4+4=0$. Express your final answers in Cartesian form.

(b) Use your answer to (a) to express the polynomial $P(z)=(z-2)^4+4$ as a product of two quadratics with real coefficients.

(3) (GI:1) Consider the following definition.

Definition. *Let $z, w \in \mathbb{C}$. We say w is the multiplicative inverse of z when $z \cdot w = 1$. When w is multiplicative inverse of z we write $z^{-1} = w$.*

(a) Let $z = 1+3i$. Use the definition of multiplicative inverse to find z^{-1}. As part of your solution you will need to solve a system of two equations and two unknowns.

(b) Let $z = a+bi$ with $z \neq 0$. Use the definition of multiplicative inverse to find an expression for z^{-1} as a function of a and b. As part of your solution you will need to solve a system of two equations and two unknowns. The work you do in this part will be very similar to the work you did in part (a).

(c) Let $z = 1+3i$. Using the complex conjugate, find values $c, d \in \mathbb{R}$ such that $c+di = \frac{1}{z}$.

(d) Let $z = a+bi$, with $z \neq 0$. Using the complex conjugate, find values $c, d \in \mathbb{R}$ that $c+di = \frac{1}{z}$. The work you do in this part will be very similar to the work you did in part (d).

(e) Using your work from the previous parts, explain how you know that for any complex number $z \neq 0$, the following identity holds:

$$z^{-1} = \frac{1}{z}.$$

(4) (GI:0) Sketch a picture of the Complex plane that shows the points contained in the set

$$\{z \in \mathbb{C} \mid |z| \leq 2\} \cap \{z \in \mathbb{C} \mid \tfrac{\pi}{6} \leq \operatorname{Arg}(z^2) \leq \tfrac{\pi}{3}\}.$$

Marking Scheme

Part B

(2)(a)

- +1 Viable approach
- +1 Final answers are correct and stated in Cartesian form
- +1 Solution written clearly, in full sentences, at a level could be understood by a fellow student

(2)(b)

- +1 Viable approach
- +1 Final answer is correct (based on final answer from (a))
- +1 Solution written clearly, in full sentences, at a level could be understood by a fellow student

(3)(a)

- +1: Correct approach using definition
- +1: Correct answer

(3)(b)

- +1: Correct approach using definition
- +1: Correct answer
- +1: Solution written clearly, in full sentences, at a level could be understood by a fellow student

(3)(c)

- +1: Correct approach using complex conjugate
- +1: Correct answer

(3)(d)

- +1: Correct approach using complex conjugate
- +1: Correct answer
- +1: Solution written clearly, in full sentences, at a level could be understood by a fellow student

(3)(e)

- +1: Correct explanation

(4)

- +1 First set drawn correctly
- +2 Second set drawn correctly